Entanglement and quantum discord dynamics of two atoms under practical feedback control

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We study the dynamics of two identical atoms resonantly coupled to a single-mode cavity under practical feedback control, and focus on the detection inefficiency. The entanglement is induced to vanish in finite time by the inefficiency of detection. Counterintuitively, the asymptotic entanglement and quantum discord can be increased by the inefficiency of detection. The noise of detection triggers control field to create entanglement and discord when no photon are emitted from the atoms. Furthermore, sudden change happens to the dynamics of entanglement.

I. INTRODUCTION

Correlations are crucial to information science. There have been a lot of studies on entanglement, a special quantum correlation, because many quantum information processes depend on entanglement [1]. Recently, new studies show that separable states can speed up some computational task compared to classical computation [2]. A more general quantum correlation, quantum discord, has also received a great deal of attention [3–9].

The crucial quantum properties can be destroyed by the influence of the environment. Many studies were done on this subject [10–15]. And many interesting phenomena have been found, such as entanglement sudden death [10], entanglement revival [11], and sudden change for quantum discord [13, 14]. In order to avoid or delay the influence of the environment, many methods have been proposed, such as quantum error correction [16, 17], decoherence-free subspace [18–20], and dynamical control [21–23]. Among them, quantum feedback control [24] is believed to be a promising method. For a system consisting of two two-level atoms coupled to a single-mode cavity which is heavily damped, the entanglement of the steady state can be improved by Markovian feedback control[25, 26]. Effects of different feedback Hamiltonians and detection processes on entanglement generation were explored [27–29]. In these works, the detection is assumed to be perfect, but it is very hard to achieve in practice. The influence of the inefficiency of detection on the dynamical and asymptotic behavior remains an open question.

In the present work, we investigate the dynamics of two identical atoms resonantly coupled to a single-mode cavity under feedback control. It was thought that the inefficiency decreases the steady entanglement, but our study shows that it may also increase the steady entanglement for some initial state. For the dynamical behavior, entanglement vanishes in
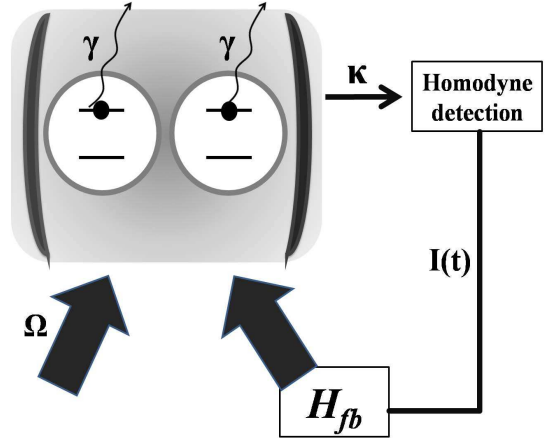

FIG. 1: Schematic view of the model. Two atoms are driven by a laser with Rabi frequency Ω, and coupled to a heavily damped cavity. The feedback Hamiltonian is applied to the atoms according to the homodyne current $I(t)$ derived from the leaky cavity.

finite time due to the inefficiency of detection. We also find that the noise of detection is an source to trigger the creation of entanglement and quantum discord. More importantly, we find that the dynamics of the entanglement may also undergo sudden change.

II. THEORETICAL FRAMEWORK

We investigate the dynamics of two identical atoms resonantly coupled to a single mode cavity which is driven by a laser field with Rabi frequency Ω and damped with decay rate κ (See FIG. 1). The atoms couple the cavity with strength g and spontaneously decay with rate γ. When the cavity mode is heavily damped, it can be adiabatically eliminated. And in the limit $\Gamma = g^2/\kappa \gg \gamma$, the dynamical evolution of this system is described by the Dicke model [26–29]:

$$\frac{d\rho}{dt} = -i[H, \rho] + D[A]\rho, \tag{1}$$

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where ρ is the density matrix of the two atoms, $H = \Omega(\sigma_x^{(1)} + \sigma_x^{(2)})$ represents the driving of the laser, and $D[A]\rho = A\rho A^\dagger - (A^\dagger A\rho + \rho A^\dagger A)/2$ represents the irreversible evolution induced by the interaction between the system and the environment with the jump operator $A = \Gamma(\sigma_-^{(1)} + \sigma_-^{(2)})$. Without losing generality, we let $\Gamma = 1$.

In this paper, we will consider the Markovian feedback [24], with the control Hamiltonian $H_{fb} = I(t)F$, where $I(t)$ is the signal from the homodyne detection of the cavity output.

In the homodyne-based scheme, the detector registers a continuous photocurrent, and the feedback Hamiltonian is constantly applied to the system. The master equation becomes [28, 29]:

$$\frac{d\rho}{dt} = -i[H + \frac{1}{2}(A^\dagger F + FA), \rho] + D[A - iF]\rho, \qquad (2)$$

where F is the feedback Hamiltonian. In this paper, we consider the symmetric feedback Hamiltonian $F = -\lambda[\mu(\sigma_x^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_x^{(2)}) + (\sigma_x^{(1)} + \sigma_x^{(2)})]$ as in [28]. It reduces to the feedback Hamiltonian in [26] when $\mu = 0$.

Practically, the efficiency of the detection, denoted by η, is less than 1. The modified master equation takes the form [30]:

$$\frac{d\rho}{dt} = -i[H + \frac{1}{2}(A^\dagger F + FA), \rho] + D[c - iF]\rho + D[\sqrt{\frac{1-\eta}{\eta}}F]\rho. \qquad (3)$$

In this paper, we choose $\Omega = 0$, $\lambda = 1$ and $\mu = 1$. For these parameters, the steady state is maximally entangled state $\phi_+ = (|eg\rangle + |ge\rangle)/\sqrt{2}$ if the initial state is in symmetric subspace for perfect detection [28]. In the basis $\{|gg\rangle, |ge\rangle, |eg\rangle, |ee\rangle\}$, if the initial state takes the form

$$\rho = \begin{pmatrix} a & 0 & 0 & e \\ 0 & c & c & 0 \\ 0 & c & c & 0 \\ e & 0 & 0 & b \end{pmatrix}, \qquad (4)$$

the form remains under evolution. This class of states includes the easily prepared state as $|gg\rangle$ and $|ee\rangle$. Since $\text{Tr}\rho = 1$, it can be found that $c = (1 - a - b)/2$.

Concurrence [31] is chosen to measure the entanglement. For density matrix in Eq. (4),

$$C(\rho) = \max[0, 1 - (\sqrt{a} + \sqrt{b})^2, 2|e| + a + b - 1].$$

Recently, a geometric measure of quantum discord (GMQD) is proposed [5], which is defined by

$$D^g(\rho) = \min_{\chi \in \Omega_0} \|\rho - \chi\|^2,$$

where Ω_0 denotes the set of zero-discord states and $\|X\|^2 = \text{Tr}X^2$ is the square norm in the Hilbert-Schmidt space. For the density matrix Eq. (4), there is a simple expression for its geometric measure of quantum discord:

$$D^g(\rho) = \min[D_1, D_2, D_3]/4,$$

where $D_1 = (-1 + a + b + 2e)^2 + (-1 + a + b - 2e)^2$, $D_2 = (a - b)^2 + (-1 + 2a + 2b)^2 + (-1 + a + b - 2e)^2$, and $D_3 = (a - b)^2 + (-1 + 2a + 2b)^2 + (-1 + a + b + 2e)^2$.

III. RESULTS

In the following, we study the dynamics of the two atoms for different initial states.

Separable initial states.—For the initial state $|ee\rangle$, the solution is

$$a(t) = \frac{e^{-2t}}{6(-2+\eta)(-1+\eta)^2}[6e^{2t}(-1+\eta)^3 - 16e^{t-\frac{t}{\eta}}(-2+\eta)\eta^2$$
$$+ e^{\frac{4t(-1+\eta)}{\eta}}(-3+\eta)\eta(1+\eta) + 3(-2+\eta)(1+\eta)(-1+3\eta)],$$

$$b(t) = e^{-2t}, \qquad e(t) = \frac{2e^{-2t}\left(-1 + e^{2t+\frac{t(-1-\eta)}{\eta}}\right)\eta}{-1+\eta}.$$

The concurrence increases in the first period and then decreases (see FIG. 2 (a)). After it decreases to zero, it increases again to steady value. More importantly, entanglement vanishes in finite time when the detection is inefficiency, $\eta < 1$. The duration of the vanishing time increases as η decreases. For the steady concurrence, we can get a closed expression as $C(\rho(t \to \infty)) = 1/(2 - \eta)$. It is an increasing function of η. When the efficiency of the detection is perfect, the steady concurrence is 1.

Since the density matrix Eq. (4) is specified by three parameters (a, b, and e), We can explain the finite-time vanishing of entanglement induced by the inefficiency of the detection by the pictorial approach, similar to [7]. In FIG. 3, separable states with the form of Eq. (4) are in the region of the wedge shape. Initial state is located on the point $(0, 1, 0)$. The final state is located on the a axis. Since the region of the valid states ($a, b \in [0, 1]$, $a + b \leq 1$ and $|e|^2 \leq ab$) is separated by the region of separable states, only when the evolution of the state just crosses the boundary of wedge shape the entanglement doesn't vanish in finite time. We can prove that it only happens when $\eta = 1$. For geometric measure of quantum discord, it follows the same tendency as concurrence, but without sudden death. It agrees with the general result that almost all states with none zero discord can never lead to states with zero discord for a finite time interval for Markovian dynamics [8]. From FIG. 2 (a) we can see that the geometric measure of quantum discord undergoes discontinuous change. For the case there is no feedback, the dynamics of the discord is different from concurrence. The concurrence remains zero all the time, while the discord increases first and then goes to zero. From FIG. 3, we can see that the evolution is confined in the region of separable states, but it doesn't remain on the straight line $a + b = 1$, $e = 0$, where discord equals zero, so the discord is not zero.

For the initial state $|gg\rangle$, the solution is given by:

$$a(t) = \frac{-1 - e^{\frac{2t(-2+\eta)}{\eta}} + \eta}{-2 + \eta}, \quad b(t) = 0, \quad e(t) = 0.$$

The concurrence and discord increases to the steady value monotonely (see FIG. 2 (b)). The dynamics is very simple. But the question is where the concurrence and discord come from. From the initial condition, we can get that the concurrence and discord are not from the initial state of the atoms,
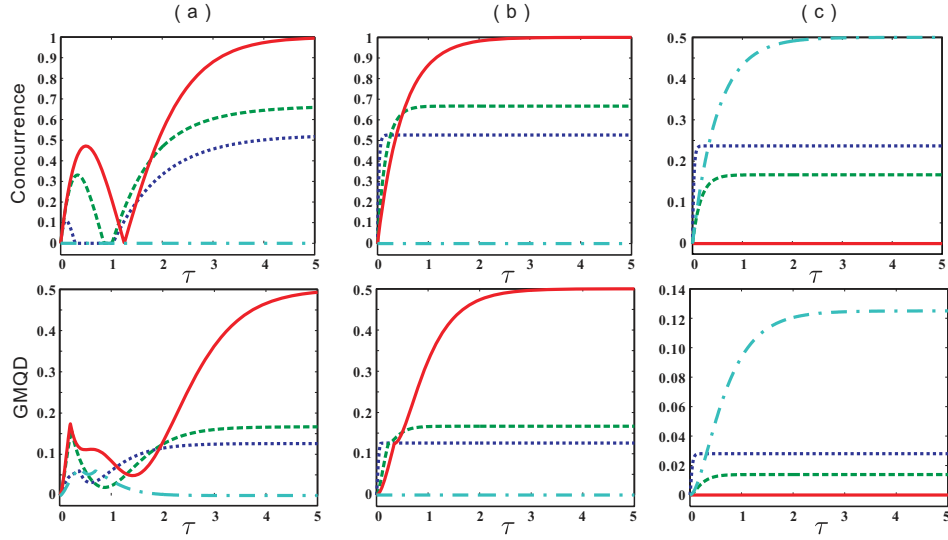
FIG. 2: (Color online) Time evolution of the concurrence and GMQD, with the initial state being (a) $|ee\rangle$, (b) $|gg\rangle$, and (c) $|eg\rangle$, all with $\lambda = 1$, $\mu = 1, \Omega = 0$, for the case of (i) $\eta = 1$ (solid curve), (ii) $\eta = 0.5$ (dash curve), (iii) $\eta = 0.1$ (dot curve), (iv) without feedback control (dot-dash curve).
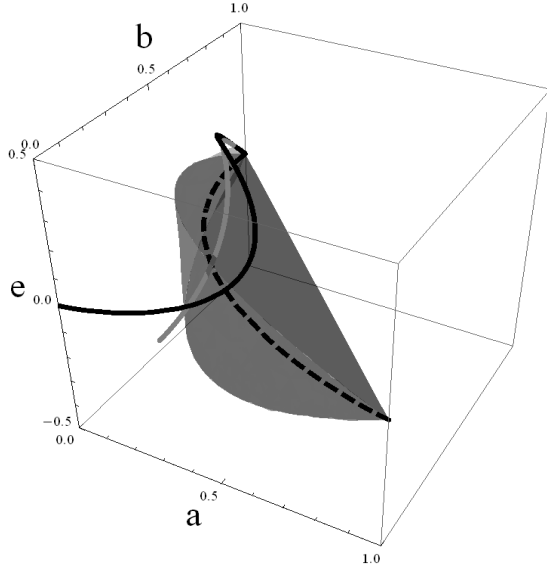

FIG. 3: (Color online) Dynamical evolution of the density matrix of two atoms with initial state $|ee\rangle$, for the case of (i) without feedback (Black dash curve), (ii) with perfect detection (Black solid curve) (iii)with imperfect detection $\eta = 0.5$ (Gray solid curve). All the separable states are in the region of wedge shape.

the environment, or the driving field. The only source of the concurrence and discord is the control field. But since the the atom are initially in the lower state, there is no photon emitted from the atoms. So the detector can not be trigged by the pho-

ton from the atoms. But this photon is not the unique source to trigger the detector. The noise can also trigger the detector, too. So we can say that the concurrence and discord is triggered by the noise.

For the initial state $|eg\rangle$, the form of the density matrix is not of the form of Eq. (4). We solve the master equation by fourth-order Runge-Kutta method. The concurrence and discord increase to steady values monotonely (see FIG. 2 (c)). But the steady-state concurrence is inverse proportional to the detection efficiency. That means the feedback is not always good for getting a high concurrence and discord. More seriously, if the detection is perfect, the concurrence and discord remains zero all the time.

Entangled initial states.—Dynamics for entangled initial states are also be For initial state $\phi_+ = (|eg\rangle + |ge\rangle)/\sqrt{2}$, the concurrence and discord decrease to steady values (See FIG. 4 (a)). And the steady-state concurrence is proportional to the detection efficiency. For the perfect detection, the concurrence remains 1 and the discord remains $1/2$ all the time. In fact, detail analysis tells us that the state remains ϕ_+. Although ϕ_+ is not a decoherence-free state under the influence of the environment, it is a steady state for the dynamics under the perfect feedback control [28]. We also study the case when the initial state is $\phi_- = (|eg\rangle - |ge\rangle)/\sqrt{2}$. Different from the case of ϕ_+, the concurrence remains 1 and discord remains $1/2$ no matter what η is (See FIG. 4 (b)). That means the inefficiency of the detection doesn't influence the dynamics.This is because, firstly, the initial state is in the decoherence free subspace, so the state of the system doesn't change under the influence of the environment, secondly, the ϕ_- is invariant under the control Hamiltonian, that means even if the feedback control is triggered by the noise, the control laser does not change the state of the system. So, we can say, the control Hamiltonian is compatible with decoherence free subspace.

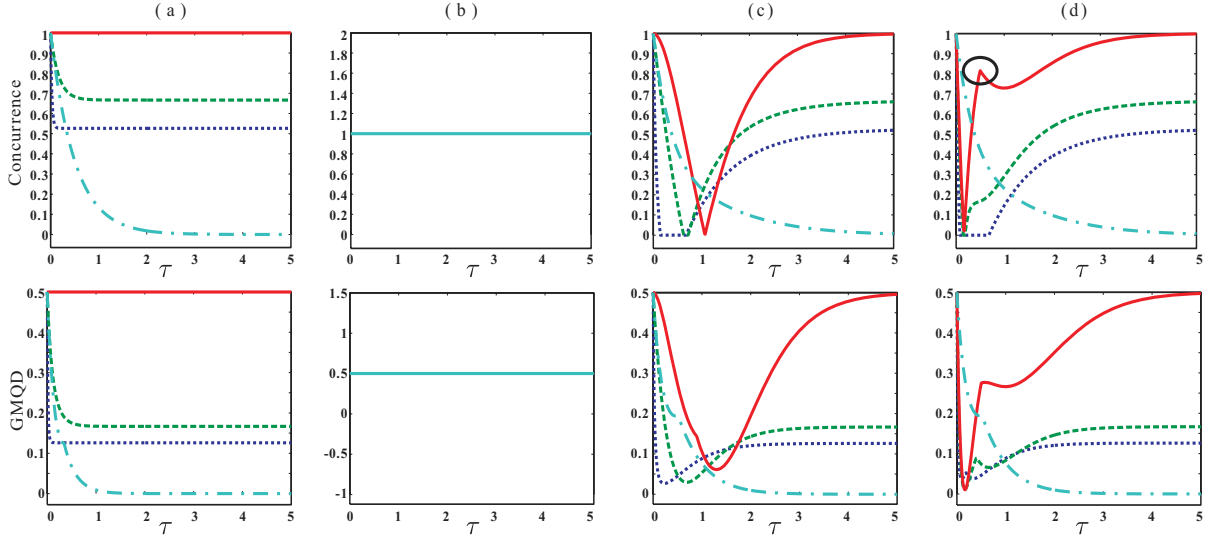For the initial state $(|ee\rangle + |gg\rangle)/\sqrt{2}$, the concurrence and

FIG. 4: (Color online) Time evolution of the concurrence and GMQD, with the initial state being (a) $(|eg\rangle + |ge\rangle)/\sqrt{2}$, (b) $(|eg\rangle - |ge\rangle)/\sqrt{2}$, (c) $(|ee\rangle + |gg\rangle)/\sqrt{2}$, and (d) $(|ee\rangle - |gg\rangle)/\sqrt{2}$, all with $\lambda = 1$, $\mu = 1$, $\Omega = 0$, for the case of (i) $\eta = 1$ (solid curve), (ii) $\eta = 0.5$ (dash curve), (iii) $\eta = 0.1$ (dot curve), (iv) without feedback control (dot-dash curve).

discord decrease at first, and then increase to steady values (See FIG. 4 (c)). And when the detection is imperfect, the concurrence vanishes in finite time. For the initial state $(|ee\rangle - |gg\rangle)/\sqrt{2}$, the concurrence and discord decrease very quickly, and then increase to steady values. And when the detection is not perfect, the concurrence vanishes in finite time (See FIG. 4 (d)). More importantly, for perfect detection, sudden change may also happen to concurrence, at $t = 0.5$. When $\eta = 1$, there is a close expression of concurrence, $C(\rho) = \text{Max}[0, C_1, C_2]$, with $C_1 = e^{-2t}[-1 + 2t - 2t^2 + e^{2t} - |1 - 2t|]$ and $C_2 = e^{-2t}|-1 + 2t| - [1 - \frac{e^{-2t}}{2} - \frac{1}{2}e^{-2t}(1 - 2t)^2]$. Sudden change happens at $t = 0.5$. But it is not the time that the size relation of C_1 and C_2 changes. Instead, at $t = 0.5$, C_1 is not continuous. This kind of sudden change is different from the sudden change of discord. Although entanglement sudden death and revival can also be regarded as a kind of sudden change happened when concurrence equals zero, in our case, the sudden change happens at a nonzero value. Since this kind of sudden change is very rare, this remains an open whether there is some physical meaning behind it or it is just from the definition of concurrence.

IV. CONCLUSION

To summarize, we study a model of two collectively damped atoms under practical feedback control. We focus on the effect of the detection inefficiency on the dynamical and asymptotic behavior of the entanglement and quantum discord. We find that the inefficiency of detection induces the entanglement to vanish in finite time, and can counterintuitively increase the asymptotic entanglement and quantum discord. The noise of detection can trigger control field to create entanglement and quantum discord when no photon are emitted from the atoms. More importantly, we find that the dynamics of entanglement also presents sudden change.

ACKNOWLEDGMENTS

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